CANDIDATE'S NUMBER

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Tests
D-2
Math 30
1968

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HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1968

MATHEMATICS 30 (NEW)

Time — 2 hours.

Note — The total time allowed for this paper is 2 hours. Distribute this time to best advantage by reading the paper, answering the questions, and reviewing the work already done. DO NOT spend too much time on any one question.

Total Possible Mark — 107.

Knott's Mathematical Tables may be used and these tables will be supplied by the Presiding Examiner. Candidates are NOT permitted to use slide rules.

SECTION A

Section A of the paper contains questions 1 - 27 inclusive. Answers to these questions are to be recorded on the separate ANSWER SHEET which will be given to you by the Presiding Examiner. Each question has four suggested answers of which only one is correct. Select the correct answer in each case and record your choice on the separate ANSWER SHEET provided as shown in the example below. There is no penalty for incorrect answers.

EXAMPLE
The product of 8 and 12 is

ANSWERS
A. 20
B. 96
C. 48
D. 72

ANSWER SHEET

A	B	C	D	E
☐	■	☐	☐	☐

DISREGARD COLUMN "E" ON ANSWER SHEET

You are to use ORDINARY HB PENCIL. Make certain your answer marks are heavy and black. If you change your mind about an answer, be sure to completely erase your first mark. There should be only one answer marked for each question. The ANSWER SHEET will be machine scored; consequently, candidates should insure that no stray pencil marks appear on the ANSWER SHEET. Write your name and school in the space provided on the ANSWER SHEET; but DO NOT write your name on this booklet.

SECTION B

Section B of this paper consists of questions 28 - 41 inclusive. Complete solutions for each question should be shown in the spaces provided in the booklet. Some marks may be obtained for partial solutions. DO NOT put work which is to be read by the sub-examiner in the ROUGH WORK ONLY column. Finished work in this section may be done in either pencil or ink (blue or black).

DO NOT FOLD EITHER THE ANSWER SHEET OR THE TEST BOOKLET.

Place your answer sheet **BEHIND** the test booklet and **INSERT BOTH IN THE SAME ENVELOPE** in such a way that the **UPPER LEFT-HAND CORNER OF THE BOOKLET** (Candidate's number) shows through the **SLASHED CORNER** of the answer envelope.

BE SURE TO SEAL THE ENVELOPE.

Machine Score
Book Score
TOTAL

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CURR HIST

SECTION A

Questions 1 - 27 inclusive have a value of 2 marks each. In each case, select the correct answer and record your choice on the separate answer sheet that has been provided.

1. Assuming that the replacement set for the variables is the real numbers for which the expression is defined, choose the appropriate quantifiers to convert the following sentence into a true statement: $x\sqrt{\dfrac{y^2}{z^4}} = \dfrac{xy}{z^2}$

A. $\forall x, \forall y, \forall z$
B. $\forall x, \forall y, \exists z$
C. $\forall x, \exists y, \exists z$
D. $\exists x, \exists y, \exists z$

2. The graph of the function defined as $f : x \rightarrow |x + 2|$ is symmetrical with respect to the

A. x-axis
B. y-axis
C. line whose equation is $x = -2$
D. line whose equation is $y = x$

3. If $f = \{(x, y) \mid y = 5x - 7, x \in R\}$, calculate $f\,[f^{-1}\,(\frac{1}{2})]$

A. $-\dfrac{9}{2}$
B. $\dfrac{1}{2}$
C. 2
D. $-\dfrac{59}{2}$

4. For the function $q : x \rightarrow 3x^2 - 2x + 1, x \in R$, determine the range

A. $\{y \mid y \leqq \frac{2}{3}, y \in R\}$
B. $\{y \mid y \geqq \frac{8}{9}, y \in R\}$
C. $\{y \mid y \leqq \frac{8}{9}, y \in R\}$
D. $\{y \mid y \geqq \frac{2}{3}, y \in R\}$

5. If $f : x \rightarrow 5^x, x \in R$, the defining equation of f^{-1} is

A. $y = \log_5 x, x \in R$
B. $y = 5^y, y \in {}^+R$
C. $y = \log_x 5, x \in R$
D. $y = \log_5 x, x \in {}^+R$

FOR ROUGH WORK
(No marks given for work in this space)

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QUAECUMQUE VERA

6. Compute: $b(2; 3, \tfrac{1}{2})$

A. $\frac{1}{4}$

B. $\frac{3}{8}$

C. $\frac{3}{16}$

D. 3

7. Given that $\begin{cases} b_1 = 1 \\ b_{n+1} = \left(\dfrac{n+1}{n+2}\right) b_n \end{cases}$, the first four terms of the sequence are

A. $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}$

B. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

C. $1, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

D. $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

8. If \$1000 is invested each year in an annuity which pays interest at 5% per annum, compounded semi-annually, after 8 deposits the amount of the annuity is

A. $\dfrac{1000\,(1.05^8 - 1)}{1.05 - 1}$

B. $\dfrac{1000\,(1.025^8 - 1)}{1.025 - 1}$

C. $\dfrac{1000\,(1.05^{16} - 1)}{1.05 - 1}$

D. $\dfrac{1000\,(1.025^{16} - 1)}{1.025^2 - 1}$

9. Solve for k: $2^1 + 2^2 + 2^3 + 2^4 + \ldots + 2^k = 126$.

A. 6

B. 5

C. 7

D. 8

10. Expand: $\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2$

A. $1 + \frac{1}{2}\sqrt{3}\,i$

B. $\frac{1}{2} - \frac{1}{2}\sqrt{3}\,i$

C. $-\frac{1}{2} - \frac{1}{2}\sqrt{3}\,i$

D. $1 - \frac{1}{2}\sqrt{3}\,i$

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FOR ROUGH WORK
(No marks given for work in this space)

11. Express in the standard form $a + bi$: $\dfrac{3 + 4i}{3 - 4i}$

A. $1 + \dfrac{24}{25}i$
B. -1
C. $-\dfrac{7}{25} + \dfrac{24}{25}i$
D. 1

12. Write the standard form of the equation of the circle with centre (2, –3) and on the point (4, –1).

A. $(x - 2)^2 + (y + 3)^2 = 2\sqrt{2}$
Geo B. $(x + 2)^2 + (y - 3)^2 = 2\sqrt{2}$
C. $(x + 2)^2 + (y - 3)^2 = 8$
D. $(x - 2)^2 + (y + 3)^2 = 8$

13. Assuming that the parabola is defined by an equation of the form $y^2 = 4px$, find the equation of the parabola with directrix defined by the equation $5x - 3 = 0$.

A. $y^2 = \dfrac{20}{3}x$
B. $y^2 = -\dfrac{12}{5}x$
Geo C. $y^2 = \dfrac{12}{5}x$
D. $y^2 = 4x$

14. Determine the co-ordinates of the foci of the hyperbola defined by the equation $9x^2 - 25y^2 = -225$.

A. $(0, \pm\sqrt{34})$
B. $(\pm\sqrt{34}, 0)$
Geo C. $(\pm 4, 0)$
D. $(0, \pm 4)$

15. Determine the equation of the hyperbola which has the line defined by $y = -\frac{3}{4}x$ as an asymptote and is on $P(5, 2)$.

A. $16x^2 - 9y^2 = 364$
B. $16x^2 - 9y^2 = -364$
Geo C. $9x^2 - 16y^2 = 161$
D. $9x^2 - 16y^2 = -161$

16. A ship carries 5 different signal flags. How many different signals can be sent using no more than 3 flags at one time?

A. 6000
B. 155
15 C. 23
D. 85

FOR ROUGH WORK

(No marks given for work in this space)

17. Calculate the number of different arrangements of the word CENTENNIAL using all the letters each time.

A. 7 !
B. $\dfrac{10\,!}{2\,!\ \ 3\,!}$
⑮ C. 5 ! × 2 ! × 3 !
D. 7 ! × 2 ! × 3 !

18. If $x = 1 + \sqrt{-2}$, evaluate $x^2 - 2x + 3$.

A. 4
B. $4 + 4i\sqrt{2}$
C. $4i\sqrt{2}$
D. 0

19. Find the 17th term in the expansion $\left(3a - \dfrac{1}{a}\right)^{20}$.

A. $-\binom{20}{17} \times 3^3a^{-14}$
B. $\binom{20}{16} \times 3^2a^{-16}$
⑮ C. $\binom{20}{17} \times 3^3a^{-14}$
D. $\binom{20}{16} \times 3^4a^{-12}$

20. Find the numerical coefficient of the middle term in the expansion of $(3x^2 - y)^{10}$.

A. $-\binom{10}{5} \times 3^5$
⑮ B. $\binom{10}{4} \times 3^4$
C. $\binom{10}{5} \times 3^5$
D. $\binom{10}{6} \times 3^4$

21. A student council consists of 3 girls and 2 boys. How many committees of 3 can be selected?

A. 60
⑮ B. 10
C. 13
D. 6

22. In how many ways can 10 girls be seated at a table if positions are indistinguishable?

⑮ A. 9 !
B. 10 !
C. 9 ! ÷ 2
D. 10 ! ÷ 2

FOR ROUGH WORK

(No marks given for work in this space)

23. A bag contains 2 white, 4 blue and 6 red marbles. If 1 marble is drawn from the bag, what is the probability that it is not blue?

A. $\frac{2}{3}$

B. $\frac{1}{3}$

16 C. $\frac{1}{8}$

D. $\frac{1}{4}$

24. If the letters of the word ALGEBRA are placed at random in a row, what is the probability that two successive letters will be 'A'?

A. 6 !

B. $\frac{2}{7}$

15 C. $\frac{1}{7}$

D. $\frac{1}{7!}$

25. A single die is thrown three times. What is the probability that all three numbers thrown are 3's?

A. $\frac{1}{72}$

16 B. $\frac{1}{120}$

C. $\frac{1}{216}$

D. $\frac{1}{2}$

26. A lottery is selling 10 tickets and offering 3 prizes. What is the probability of drawing at least one prize with 4 tickets?

A. $\frac{1}{30}$

$^{10}C_3 = \frac{10 \times 9 \times 8 \times 7!}{3!\,7!} = 120$

16 B. $\frac{5}{6}$

$\frac{4}{120}$

C. $\frac{2}{15}$

D. $\frac{17}{30}$

27. Two dice, 1 red, 1 white, are tossed. If the red die turns up 6, what is the probability of obtaining a total of 11?

A. $\frac{1}{18}$

16 B. $\frac{1}{2}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

FOR ROUGH WORK

(No marks given for work in this space)

Values

SECTION B

ROUGH WORK ONLY
(No marks given for work in this space)

Complete solutions for questions 28 - 41 must be shown in the spaces provided in the booklet.

3 **28.** Let f and g be functions defined by:

$$f(x) = \frac{x}{x+1},\ x \in R,\ x \neq -1;$$

$$g(x) = \frac{1}{x-1},\ x \in R,\ x \neq 1$$

Compute $f\left[g\left(\frac{1}{x}\right)\right]$

3 **29.** Determine, in mapping notation, the function whose graph is described as a straight line on $P(-3, 1)$ and $Q(2, -5)$.

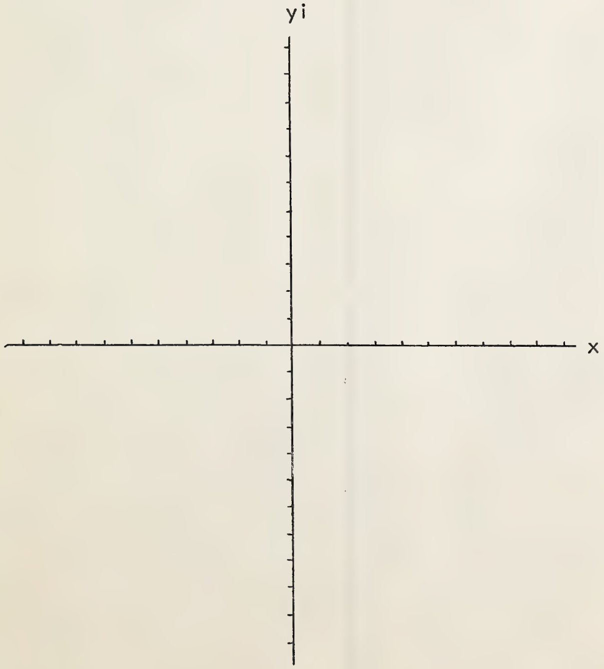
Values

3 **30.** Prove that the sequence f, such that $f = 3 \cdot 2^{n-1}$ is geometric.

ROUGH WORK ONLY
(No marks given for work in this space)

3 **31.** Sketch the graph of the set

$$A = \left\{ z \,\middle|\, | z - 3 + 2i | = 4, z \in C \right\}$$



Values

3 **32.** Solve: $\frac{z-1}{z+1} = \frac{2z}{z+5}$, $z \in C$

3 **33.** Prove: $\frac{(n-1)!}{(n-6)!\,5!} + \frac{(n-1)!}{(n-5)!\,4!} = \binom{n}{5}$

ROUGH WORK ONLY
(No marks given for work in this space)

Values

3 **34.** Show that $(5)_3 - (5)_2 = 2 \times (5)_2$.

4 **35.** Solve for x, $x \in R$:

$$\sqrt{x + 18} = x - 2$$

ROUGH WORK ONLY
(No marks given for work in this space)

[OVER]

Values

4 **36.** Determine the values of k for which the equation $k^2x^2 - (2k + 3)x + k + 2 = 0$, $k \neq 0$, has roots whose sum is greater than or equal to 1.

ROUGH WORK ONLY
(No marks given for work in this space)

4 **37.** Find the sum of the series

$(-8) + (-5) + (-2) + \ldots . + 49.$

Values

4 **38.** Find the sum to 10 terms of the sequence 'a' such that $a_n = 2n + 2^n$.

14

5 **39.** Three coins are tossed and it is noted that not all three fall heads or not all three fall tails. Find the probability that the number of heads is an odd number.

16

ROUGH WORK ONLY
(No marks given for work in this space)

LB 3054 C2 D419 1968 MATH-30
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Values

5 **40.** Determine, algebraically, the ordered pairs of

$$A = \{(x, y) \mid 6x + 4y = 5x^2 - 4y^2 = 16,\ x, y \in R\}$$

Values

6 **41.** Prove by mathematical induction that

$$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

Yes

ROUGH WORK ONLY
(No marks given for work in this space)

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Departmental examinations,
[grade 12] 1968. -

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